Lecture 13

Material and Homework

Read:

Chapter 10 Yariv and Yeh

Chapter 11 Yariv and Yeh

Detection of Optical Signals

- Thermal: Temperature change with photon absorption
  - Thermoelectric
  - Pyromagnetic
  - Pyroelectric
  - Liquid crystals
  - Bolometers
- Wave Interaction: Exchange energy between waves at different frequencies
  - Parametric down-conversion
  - Parametric up-conversion
  - Parametric amplification
- Photon Effects: Generation of photocarriers from photon absorption
  - Photoconductors
  - Photoemissive
  - Photovoltaics

Optical Detection and Noise

- We turn our attention to converting optical signals to electrical signals
- We want to understand the physical mechansisms that govern “photodetection” and how well the signal is recovered
- One of the primary ways a signal gets degraded is when noise gets added to the information modulation on the optical signal, so we need to understand the different noise sources and how they interact with signal detection
- Noise impacts an optical system in the following ways
    - Measurement: Noise causes random fluctuations in the detected signal and limits the smallest power that can be detected accurately. This noise is generated in the detection devices and systems.
    - Generation: Noise is present in optical sources which will interfere with the accurate measurement of optical signals using a photodetector.
    - Amplification and Transmission: There is noise in the process of trying to increase the level of a signal using optical or electrical amplifiers. The medium used to transmit signals (e.g. optical fiber) can also manifest certain types of noise.

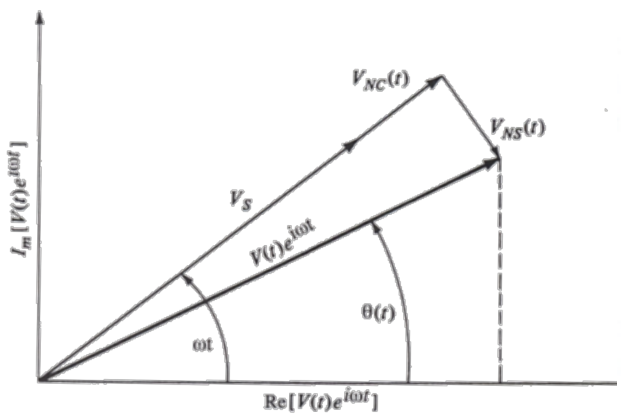
Limitations due to Noise Power

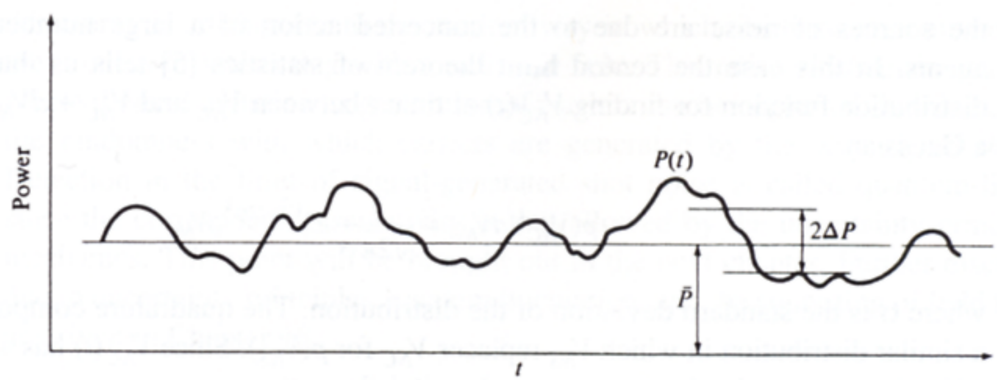
• The signal and additive noise can be represented in either the time domain or using phasor notation as shown below.

• Let V_s be the magnitude of a sinusoidally modulated signal $v_s(t)$ oscillating at ω: $v_s(t) = V_s\cos(\omega t)$.

• Let the noise be additive to the signal in both amplitude and phase and represented by an in-phase component $V_{NC}(t)$ and a quadrature component $V_{NS}(t)$. Also assume These noise components vary slowly w.r.t. cos(ωt).

• The total signal in phasor notation can be written as $v(t) = \mathrm{Re}\left\{\left[V_S + V_{NC}(t) - iV_{NS}(t)\right]e^{i\omega t}\right\}$





Effect of Noise Sources

• Since noise is due to many different random events that act independently, we can invoke the central limit theorem to describe the probability distribution for the amplitude and phase of both the real (in-phase) and imaginary (quadrature) noise components.

• This additive randomness in amplitude and phase creates a sort of "bullseye" pattern where the vector *v(t)* has a probability landing as its "average" is spinning around the imaginary plane at frequency ω.

• For a large number of random independent events, we can utilize the central limit theorem and describe the noise using Gaussian statistics with variance σ_{NS} and σ_{NC} and averages $\langle V_{NS} \rangle$ and $\langle V_{NC} \rangle$

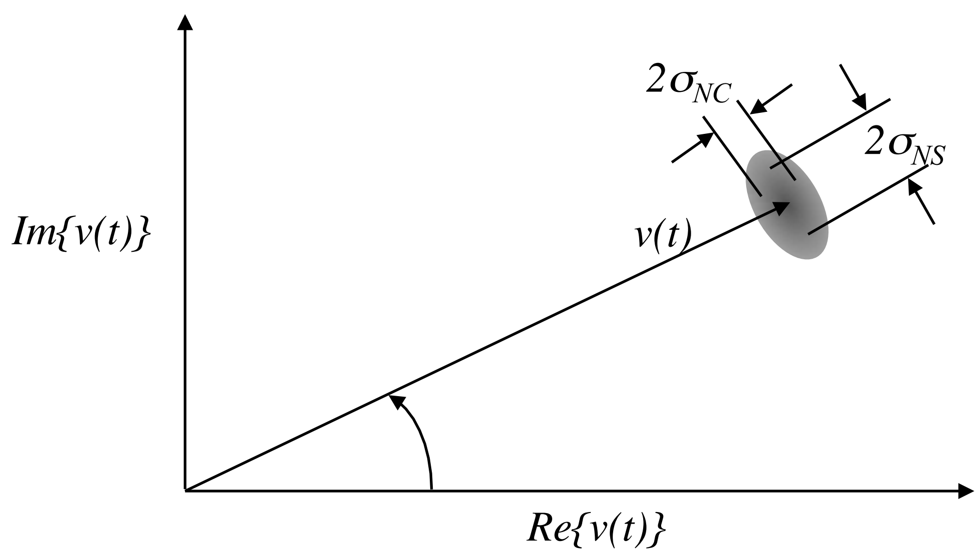
$$\rho(V_{NC}) = \frac{1}{\sqrt{2\pi}\sigma_{NC}} e^{-V_{NC}^2 / 2\sigma_{NC}^2}$$

$$\rho(V_{NS}) = \frac{1}{\sqrt{2\pi}\sigma_{NS}} e^{-V_{NS}^2 / 2\sigma_{NS}^2}$$

$$\sigma_{NC}^2 = \overline{V_{NC}^2} = \int_{-\infty}^{\infty} V_{NC}^2 \rho(V_{NC}) dV_{NC}$$

$$\sigma_{NS}^2 = \overline{V_{NS}^2} = \int_{-\infty}^{\infty} V_{NS}^2 \rho(V_{NS}) dV_{NS}$$

$$\langle V_{NC}(t) \rangle = \langle V_{NS}(t) \rangle = 0$$



Noise Power

• The average power associated with the signal and noise is given by the following, which can be reduced by assuming the variance of the in-phase and quadrature components are equal ($\sigma_{NC} = \sigma_{NS}$)

$$\begin{aligned}
\bar{P} &= \langle P(t) \rangle \\
&= \left\langle \frac{1}{2}\left(V(t)e^{i\omega t}\right)\left(V^{*}(t)e^{-i\omega t}\right)\right\rangle \\
&= \left\langle \frac{1}{2}\left(V_S^2 + 2V_S V_{NC} + V_{NC}^2 + V_{NS}^2\right)\right\rangle \\
&= \frac{1}{2}\left(\overline{V_S^2} + \overline{V_{NC}^2} + \overline{V_{NS}^2}\right) \\
&= \frac{1}{2}\left(\overline{V_S^2} + \sigma_{NC}^2 + \sigma_{NS}^2\right) \\
&= \frac{1}{2}\left(\overline{V_S^2} + 2\sigma^2\right)
\end{aligned}$$

Noise Power

• When measuring the power, there is an uncertainty due to the noise components, which can be described by a deviation in the average power ΔP(t)

$$P(t) = \overline{P} + \Delta P(t)$$

• Using the rms values

$$\Delta P = \left[\overline{\left(P(t) - \overline{P} \right)^2} \right]^{1/2}$$

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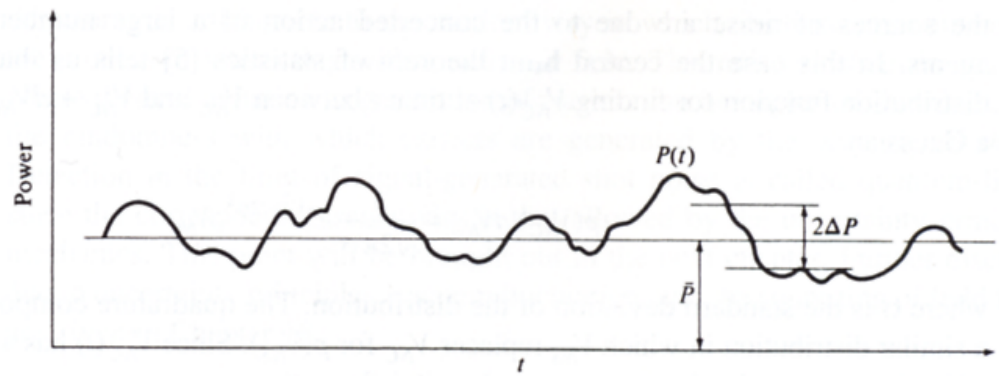
• And defining the signal power P_S as the power measured in the absence of noise and converting to Gaussian variance

$$\Delta P = \sigma \left(2P_S + \sigma^2 \right)^{1/2}$$



Noise Equivalent Power (NEP)

• The point where the signal power and noise power become equal is of interest in order to describe the statistical lower limit on detectable signal using detection without signal processing.

$$NEP = P_{\lim it} = \Delta P = P_S$$

$$NEP = P_{\lim it} = \sigma\left(2P_{\lim it} + \sigma^2\right)^{1/2}$$

$$NEP = P_{\lim it} = \sigma^2\left(1 + \sqrt{2}\right)$$

$$NEP = P_{\lim it} = P_N\left(1 + \sqrt{2}\right)$$

• Where we have defined $P_N = \sigma^2$ is sometime used as the minimum detectable power instead of the NEP.

Noise Definitions and Theorems

- For a signal v(t) and its Fourier Transform $V_T(\omega)$ observed over a time interval T/2<t<T/2, we define the average power as

$$P = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left\{ v(t)\left[\int_{-\infty}^{\infty} V_T(\omega) e^{i\omega t} d\omega\right]\right\} dt$$

$$= \frac{4\pi}{T}\int_{0}^{\infty} \left|V_T(\omega)\right|^2 d\omega$$

- And we define the spectral density function by

$$S_v(\omega) = \lim_{T\to\infty} \frac{4\pi\left|V_T(\omega)\right|^2}{T}$$

Wiener-Khintchine Theorem

• We define the autocorrelation function $C_v(\tau)$ as the time average of a signal multiplied by shifted versions of itself as a function of τ

$$C_v(\tau) = \overline{v(t)v(t+\tau)}$$

• The W-K Theorem tells us that the power spectral density and autocorrelation functions are related by the Fourier and inverse Fourier transforms

$$C_v(\tau) = \frac{1}{2}\int_{-\infty}^{\infty} S_v(\omega) e^{i\omega\tau} d\omega$$

$$S_v(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} C_v(\tau) e^{i\omega\tau} d\tau$$

Discrete Train of Random Events

- For a time-dependent random variable made up of individual events occurring at discrete times $f(t-t_i)$, the events observed over a time interval T and its Fourier transform can be described by

$$i(t) = \sum_{i=1}^{N_T} f(t-t_i), \quad 0 \le t \le T$$

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$$I_T(\omega) = \sum_{i=1}^{N_T} F_i(\omega)$$

$$F_i(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t-t_i)e^{-i\omega t}dt = \frac{e^{-i\omega t_i}}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt = e^{-i\omega t_i}F(\omega)$$

- If we take the magnitude square of the Fourier Transform

$$|I_T(\omega)|^2 = |F(\omega)|^2 \sum_{i=1}^{N_T}\sum_{j=1}^{N_T} e^{i\omega(t_i-t_j)}$$

$$= |F(\omega)|^2 \left(N_T + \sum_{i\ne j}^{N_T}\sum_{j}^{N_T} e^{i\omega(t_i-t_j)} \right)$$

Discrete Train of Random Events

• The last term averages out to zero for a large ensemble of random events

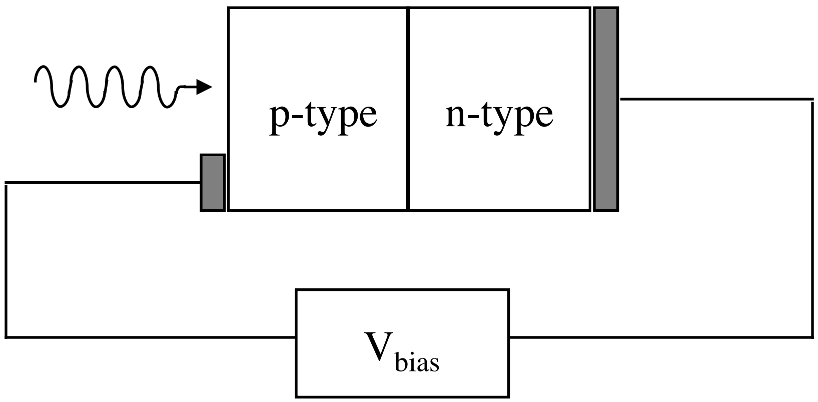
$$\left|I_T(\omega)\right|^2 = \overline{N_T}\left|F(\omega)\right|^2 = \overline{N}T\left|F(\omega)\right|^2$$

••The spectral density is defined by

$$S_i(\omega) = 4\pi\overline{N}\left|F(\omega)\right|^2$$
$$S_i(\nu) = 8\pi^2\overline{N}\left|F(2\pi\nu)\right|^2$$

• Where the result $S_i(\nu)$ is known as ***Carson's Theorem***

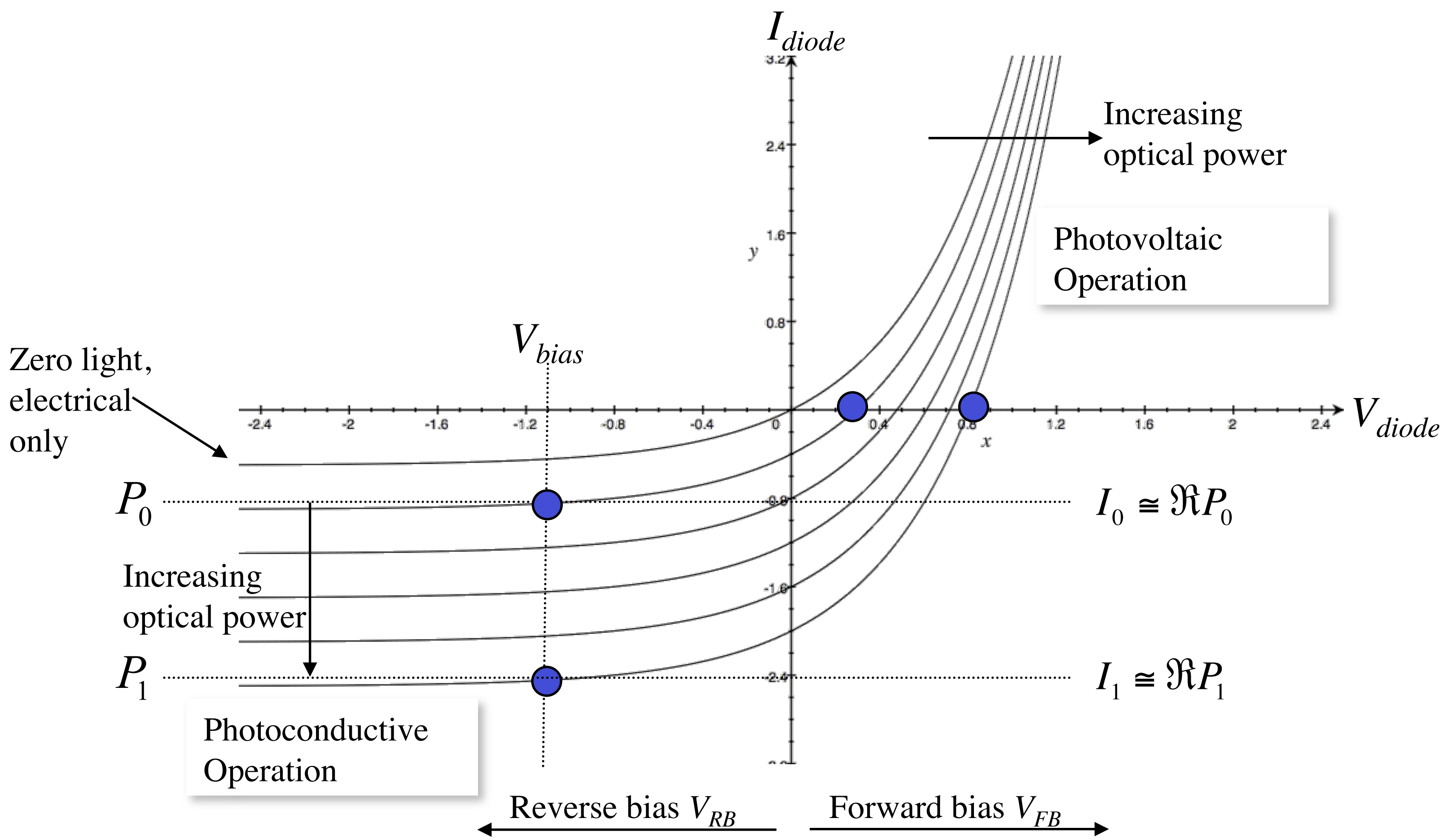
p-n Junction Photodiode Equation



$$I = (I_s)[\exp^{qV_{bias}/K_B T} - 1] - I_{photo}$$
$$= I_{dark} - I_{photo}$$

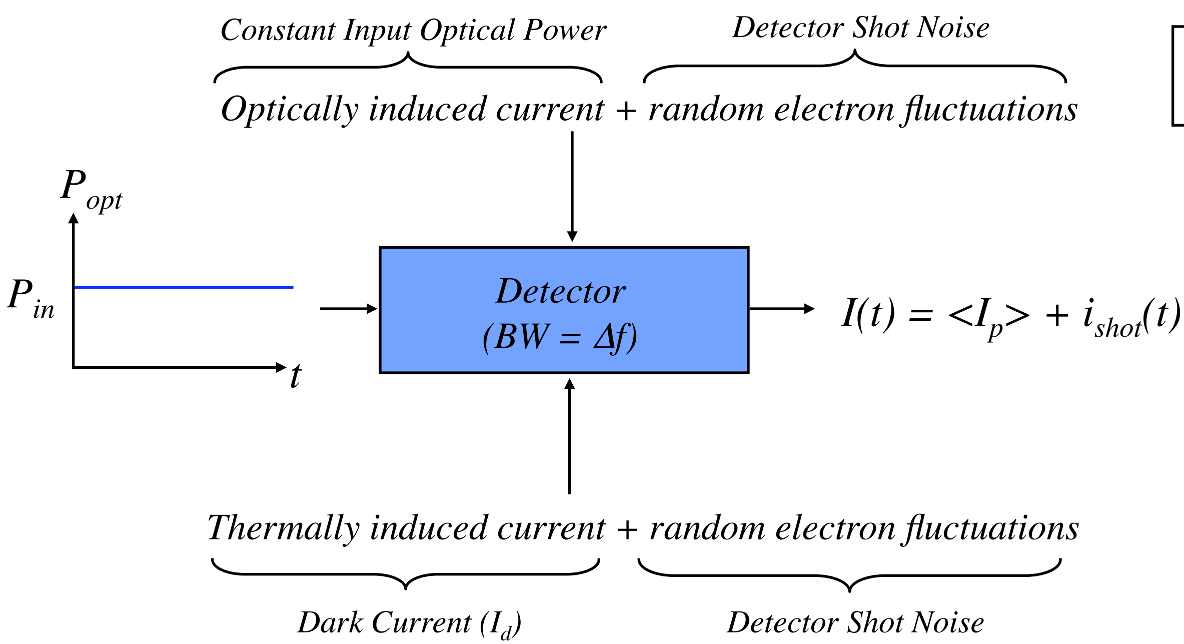
- I_{dark} = is the current that occurs with zero optical input
- $I_s = I_{th}$ is the thermal or saturation current that occurs in normal (non-illuminated) diode operating mode
- I_{photo} is photo-generated current = $\frac{\eta q}{h\nu} P_{rcvd}$
- q is the electron charge
- V_{bias} is applied bias voltage (positive = forward, negative=reverse)
- K_B is Boltzman's constant
- T is temperature (usually in Kelvin, depending on units of K_B)

p-n Junction Photodiode Regions of Operation

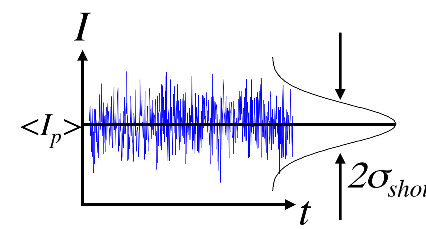


Electrical Shot Noise

- The shot noise generated in the photodetection process is physically due to the "quantum granularity" of the received (and photo converted) optical signal
- It sets the ultimate limit of an optical receiver (only in theory, as shown later)
- It is a Poisson noise, but it is usually approximated as a Gaussian noise

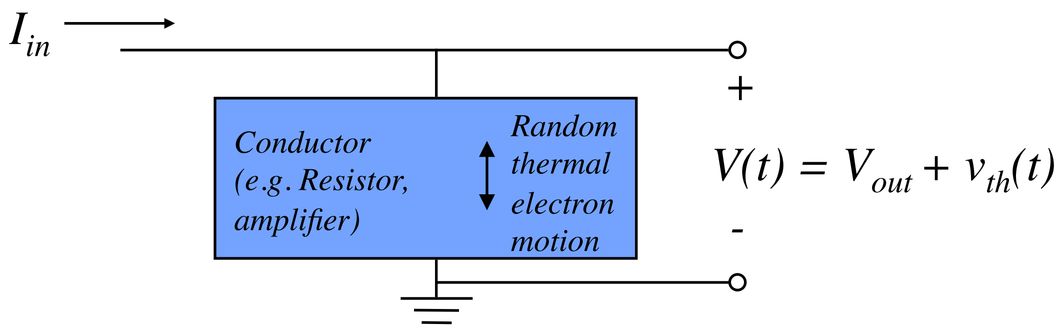


$$\boxed{\sigma^2_{Shot} = 2q(I_p + I_d)\Delta f}$$

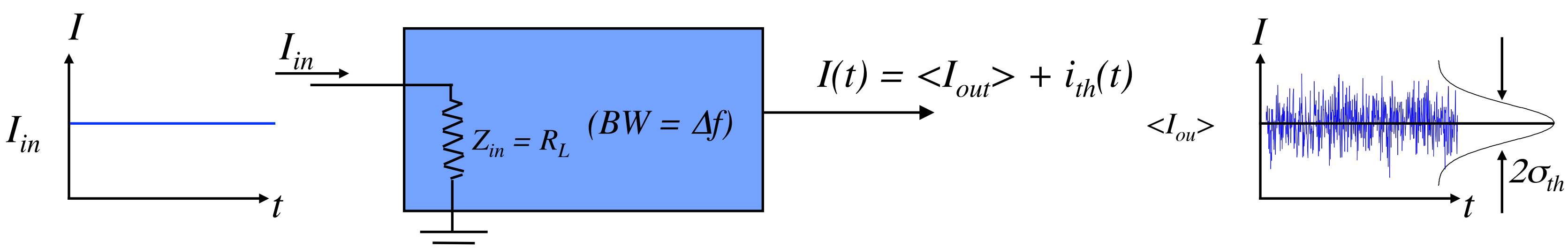


Thermally Generated Noise

- Noise generated by any electrical component due to the thermal motion of electrical carriers inside conductive media
- It is a Gaussian noise source



$$\sigma^2_{th} = 4\,(k_B T/R_L)\Delta f$$



Amplifier Noise

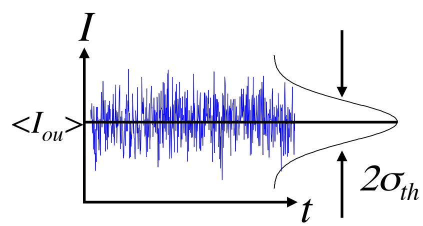
- The amplifier enhances the thermal noise at the input by a factor called the amplifier “Noise Figure

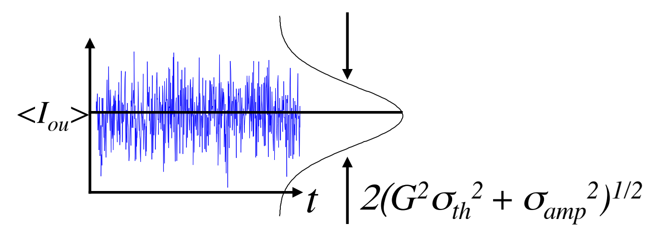
$SNR_{in} = \langle I_{in}\rangle^2/\sigma^2_{th}$

$SNR_{out} = \langle I_{out}\rangle^2/\sigma^2_{out}$

$I_{in}(t) = \langle I_{in}\rangle + i_{th}(t) \longrightarrow$ **Amplifier** *(BW = Δf)* *(Gain = G)* $\longrightarrow$

$$\begin{aligned} I_{out}(t) &= G\,[\langle I_{in}\rangle + i_{in}(t)] + i_{amp}(t) \\ &= G\,\langle I_{in}\rangle + [G\,i_{in}(t) + i_{amp}(t)] \\ &= \langle I_{out}\rangle + i_{out}(t) \end{aligned}$$





$$F_n = (SNR_{in})/(SNR_{out}) = \left(\langle I_{in}\rangle^2/\sigma^2_{th}\right)/\left(G^2\,\langle I_{in}\rangle^2/(G^2\sigma_{th}^2 + G^2\sigma_{eff}^2)\right) = \sigma^2_{out}/\,\sigma^2_{th}$$

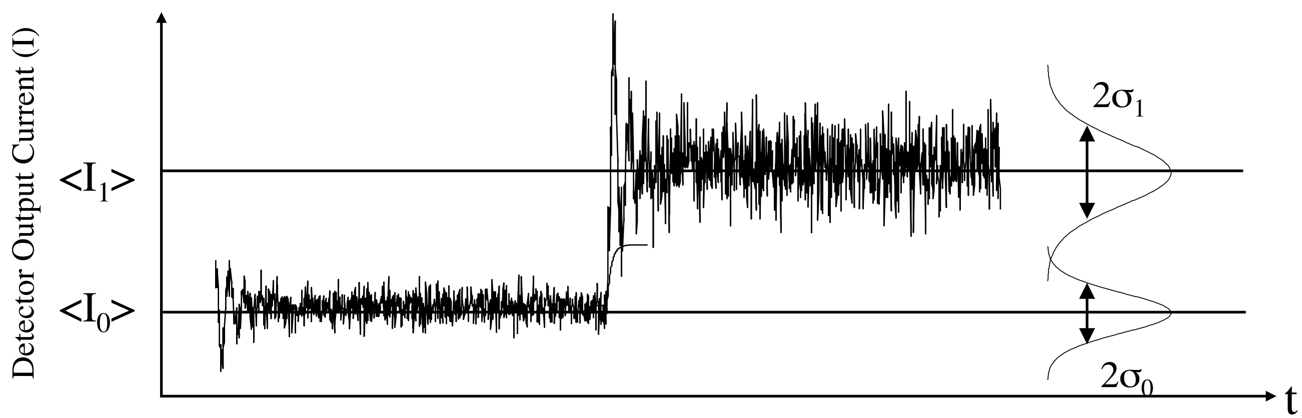
$$\sigma^2_{out} = \sigma^2_{th}\,F_n$$

$$\sigma^2_{out} = 4\,(k_B T/R_L) F_n \Delta f$$

Electrical Signal-to-Noise Ratio (SNR)

- At the receiver, there is noise on the signal arriving at the input and and after detection added to that is noise that is injected at various stages of the receiver
  - The current output of the receiver $i_n(t)$ has current contributions from
    - Electrical shot noise
    - Thermal noise
    - APD detectors have additional multiplication noise
    - Amplifier noise

$$P_{opt}(t) = P_{Sig}(t) + P_n(t) \longrightarrow \boxed{\text{Receiver}} \longrightarrow I(t) = I_p(t) + i_n(t)$$



Electrical Signal-to-Noise Ratio (SNR)

➥ At the receiver, there is noise on the signal arriving at the input and there is noise that is injected at various stages of the receiver

$$P_{opt}(t) = P_{Sig}(t) + P_n(t) \longrightarrow \boxed{\text{Receiver}} \longrightarrow I(t) = I_p(t) + i_n(t)$$

$$\sigma^2_{RIN} + \sigma^2_{MPN} + \sigma^2_{BEAT} + \sigma^2_{SIG\text{-}SPONT}$$

$$\sigma^2_{Thermal} + \sigma^2_{Shot} + \sigma^2_{BEAT} + \sigma^2_{SIG\text{-}SPONT}$$

For a pin detector. Replace with σ_{APD} if an APD is used